Fermi-Dirac statistics:-

In fermi-Dirac distribution, the particles are indinguishable and no two particles can occupy a single quantum state i.e it follows Pauli exculsion principle. So the occupation number is within zero or one. its spin is $\frac{1}{2}$ integral of $\hbar$.
Suppose, we have 'n_i' particles to be put in 'g_i' states.
The first particle can occupy 'g_i' states in 'g_i' ways, the second particle is (g_i-1) ways, the third particle is (g_i-2) ways and so on. The number of possible distinct microstates is

$$\Omega_i\{g_i, n_i\} = \frac{g_i(g_i-1)\cdots(g_i-n_i+1)}{n_i!}$$

we divide by $n_i!$ because the distribution among n_i particles themselves does not create a new state.

Thus,

$$\Omega_i\{g_i, n_i\} = \frac{g_i(g_i-1)\cdots(g_i-n_i+1)(g_i-n_i)\cdots 1}{(g_i-n_i)!\, n_i!}$$

$$= \frac{g_i!}{(g_i-n_i)!\,n_i!} \quad (g_i \geq n_i)$$

If there are K possible distinct groups, we have

$$\Omega_i\{g_i, n_i\} = \prod_{i=1}^{K} \frac{g_i!}{(g_i-n_i)!\,n_i!} \quad (g_i \geq n_i)$$

To obtain the fermi-Dirac distribution function, we maximize $\Omega\{g_i, n_i\}$ with respect to n_i subject to the condition that the

Total No. of particles $(N) = \sum n_i$ = constant
and total energy $(U) = \sum \epsilon_i n_i$ = constant

Thus,

$$\ln \Omega \{g_i, n_i\} = \sum_i (\ln g_i! - \ln n_i!) - \sum_i \ln (g_i - n_i)!$$

$$= \sum_i g_i \ln g_i - g_i - n_i \ln n_i + n_i + n_i - (g_i - n_i) \ln (g_i - n_i) + g_i - n_i$$

$$= \sum_i g_i \ln g_i - n_i \ln n_i - (g_i - n_i) \ln (g_i - n_i) \quad \text{--- (1)}$$

We maximize with respect to n_i

$$\therefore \delta \ln \Omega (g_i, n_i) = \sum_i [-\ln n_i + \ln (g_i - n_i)] \delta n_i = 0$$

Using Langrange's undetermind multipliers, we get

$$\sum_i [-\ln n_i + \ln (g_i - n_i) - (\alpha + \beta \epsilon_i)] \delta n_i = 0$$

since δn_i is arbitrary, we get

$$\ln \left[\frac{(g_i - n_i)}{n_i} \right] = \alpha + \beta \epsilon_i$$

$$\therefore n_i = \frac{g_i}{e^{\alpha + \beta \epsilon_i} + 1}$$

$$\text{Thus, } f(\epsilon_i) = \frac{n_i}{g_i} = \frac{1}{e^{\alpha + \beta \epsilon_i} + 1} \quad \text{--- (2)}$$

This is Fermi-Dirac distibution function.

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